Sample problems for 4th exam

1. A family has four children. Let X be the number of girls.

i) Find all the possible outcomes for the gender of these children.

ii) Compute $E(X)$

iii) Compute the standard deviation of X.

(*A problem where we do a basic study of a random variable stemming from an experiment of luck.*)

Solution: i) We have sixteen possible outcomes:

BBBB BBBG BBGB BBGG

BGBB BGBG BGGB BGGG

GBBB GBBG GBGB GBGG

GGBB GGBG GGGB GGGG

ii) The table of values of X and their corresponding probabilities is:

k	$P(X=k)$
0	1/16
1	4/16
2	6/16
3	4/16
4	1/16

So

$$E(X) = \frac{32}{16} = 2$$

iii) In addition to $E(X)$, we will need $E(X^2)$. The corresponding table is:

k	$P(X = k)$
0	1/16
1	4/16
4	6/16
9	4/16
16	1/16

So

$$E(X) = \frac{80}{16} = 5$$

This means that $s = \sqrt{E(X^2) - (E(X))^2} = \sqrt{5-4} = 1$.

2. Compute the mean and the standard deviation of the sample:

$$1, 0, 2, 3, 1$$

(*A problem where you will be asked to find the mean and the standard deviation of a sample.*)

Solution: The sample mean is:

$$\overline{X} = \frac{1+0+2+3+1}{5} = 1.4$$

We will use the computational formula of the standard deviation, which is:

$$s = \sqrt{\frac{\sum X^2 - n\left(\overline{X}\right)^2}{n-1}}$$

where n is the size of the sample and $\overline{X}$ is the sample mean.

We perfom the computation according to the following table:

X	X^2
1	1
0	0
2	4
3	9
1	1

The right column adds up to 15. So the numerator of the fraction will be:

$$15 - 5 \cdot (1.4)^2$$

which is 5.2. Therefore the standard deviation will be:

$$\sqrt{\frac{5.2}{4}} = 1.14$$

3. It is given that the average height of a particular age group is 50 inches with standard deviation 3 inches. A given child has height 48 inches. Compute the child's percentile.

A problem where you compute the percentile based on the z-score.

Solution: The z-score of the specific child is:

$$\frac{48 - 50}{3} = -.66$$

The tail of .66 in the z-score tables is .2546. (Note that since the z-score is negative, we are looking up the tail). Therefore the child is in the 25th percentile of height of their age group.

4. It is known that height is normally distributed with average 5.8 feet and standard deviation .3 feet. We would like to test the hypothesis that the average height of a certain small town is actually larger than 5.8 feet. Suppose that we have collected a sample of 40 heights from that small town and that the average is 5.85. If μ is the average height in that smallt town, determine whether $\mu > 5.8$ with a confidence level of 95%.

(*A problem where you will be asked to test a hypothesis by using the one-tailed z-score.*)

Solution: We begin by giving the research hypothesis and the null hypothesis. The research hypothesis is:

$$H_1 : \mu > 5.8$$

and the null hypothesis is:

$$H_o : \mu \leq 5.8$$

We are using alpha level .05 and the corresponding critical value is 1.65. The sample standard deviation is

$$\frac{.3}{\sqrt{40}} = 0.047$$

The z-score of our sample average is:

$$\frac{5.85 - 5.8}{0.047} = 1.06$$

Therefore the given z-score is smaller than the critical value and we fail to reject the null hypothesis.

5. It is known that a US family has an average of 2.6 children. It is speculated that this average is different among families in Alaska. We collect a sample of 10 families with average 2.3 chidren and standard deviation .7 children. Determine whether the hypothesis is sound with confidence level 95%.

(*A problem where you will be asked to test a hypothesis by using the two-tailed T-test.*)

Solution: We will use the T-test. We begin by giving the two hypotheses. The research hypothesis is:

$$H_1 : \mu \neq 2.6$$

and the null hypothesis is:

$$H_o : \mu = 2.6$$

We are using alpha level .05 and we have 9 degrees of freedom. From the T-table, we determine that the critical value is 2.26. Since we are dealing with the two tailed situation, this means that we will reject the null if the T-score is greater than 2.26 or smaller than -2.26.

The sample standard deviation can be taken to be $.7/\sqrt{10} = 0.221$. Then the T-score is given by:

$$\frac{2.3 - 2.6}{.221} = 1.35$$

By the observations that we made above, we fail to reject the null hypothesis.

6. It is conjectured that the 1st book of an author is usually shorter than the second book. We collect the following data on the number of pages of the 1st and the 2nd book of 7 different authors:

123	134
95	90
220	270
300	295
100	120
150	172
100	111

Determine with a confidence level of 95% whether this hypothesis is sound or not.

(*A problem where you will be asked to test a hypothesis for related samples by using the one-tailed T-test.*)

Solution: We are dealing with the case of related samples. We begin by considering the differences between the numbers in the two samples:

$$-11, 5, -50, 5, -20, -22, -11$$

The average is computed to be -14.85 and the standard deviation 18.84.

The research hypothesis is:

$$H_1 : \mu_{1st} < \mu_{2nd}$$

and the null hypothesis is:

$$H_o : \mu_{1st} \geq \mu_{2nd}$$

We are using alpha level .05 and we have 6 degrees of freedom. From the T-table, we determine that the critical value is 1.94. Since we are dealing with the one tailed situation, this means that we will reject the null if the T-score is smaller than -1.94.

The sample standard deviation can be taken to be $18.84/\sqrt{7} = 7.12$. Then the T-score is given by:

$$\frac{-14.85}{7.12} = -2.08$$

By the observations that we made above, we reject the null hypothesis.

7. It is known that the score of a standardized exam is normally distributed with average 69 and standard deviation 4. We would like to test the hypothesis that the average score among the students of a local school is actually different from 69. Suppose that we have collected a sample of 40 scores from that school and that the average is 67. If μ is the average score in that local school, determine whether $\mu \neq 69$ with a confidence level of 95%.

(*A problem where you will perform two-tailed hypothesis testing based on the z-score.*)

Solution: We begin by giving the research hypothesis and the null hypothesis. The research hypothesis is:

$$H_1 : \mu \neq 69$$

and the null hypothesis is:

$$H_o : \mu = 69$$

We are using alpha level .05 and the corresponding critical values is ± 1.96. The sample standard deviation is

$$\frac{4}{\sqrt{40}} = 0.632$$

The z-score of our sample average is:

$$\frac{67 - 69}{0.632} = -3.16$$

Therefore the given z-score is smaller than the negative of the two critical values and we proceed to reject the null hypothesis.